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Higher derivative operators from Scherk-Schwarz supersymmetry breaking on $\mathcal{T}^2/Z_2$.

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Abstract: In orbifold compactifications on $\mathcal{T}^2/Z_2$ with Scherk-Schwarz supersymmetry breaking, it is shown that (brane-localised) superpotential interactions and (bulk) gauge interactions generate at one-loop higher derivative counterterms to the mass of the brane (or zero-mode of the bulk) scalar field. These brane-localised operators are generated by integrating out the bulk modes of the initial theory which, although supersymmetric, is nevertheless non-renormalisable. It is argued that such operators, of non-perturbative origin and not protected by non-renormalisation theorems, are generic in orbifold compactifications and play a crucial role in the UV behaviour of the two-point Green function of the scalar field self-energy. Their presence in the action with unknown coefficients prevents one from making predictions about physics at (momentum) scales close to/above the compactification scale(s). Our results extend to the case of two dimensional orbifolds, previous findings for S^1/Z_2 and $S^1/(Z_2 \times Z_2')$ compactifications where brane-localised higher derivative operators are also dynamically generated at loop level, regardless of the details of the supersymmetry breaking mechanism. We stress the importance of these operators for the hierarchy and the cosmological constant problems in compactified theories.

Keywords: Beyond Standard Model, Supersymmetry Breaking, Large Extra Dimensions, Field Theories in Higher Dimensions.

Contents

1. Introduction

In recent years models for physics beyond the Standard Model with additional space dimensions [1] have drawn much attention from the physics community. In particular the supersymmetric versions of such models have been especially popular due to the possibility of understanding the mechanism of transmitting the supersymmetry breaking in a geometric way [2 − 5] and of generating radiative electroweak symmetry breaking due to extra dimensions [6].

If supersymmetry is assumed to be broken in a hidden sector.[2] That is geometrically separated in extra dimensions or that does not couple directly to the visible sector, then there are no soft mass terms at the tree level, in the visible sector[3] If so, non-zero soft mass parameters are nevertheless generated by the loop corrections, which provide in such case the leading contribution. Further, it has been shown [3, 9] that the codimension-one localised sources of supersymmetry breaking on orbifolds are equivalent to the Scherk-Schwarz breaking due to non-trivial boundary conditions. Then, in the presence of either local or Scherk-Schwarz breaking of supersymmetry, one can compute the loop-corrections to the masses of the scalar fields in the visible sector. As an application, in some models with one extra dimension compactified on S^1/Z_2 or $S^1/(Z_2 \times Z_2')$ orbifolds, the one-loop correction to the mass of a scalar (Higgs) field may be UV cutoff independent and have a negative sign to trigger (electroweak) symmetry breaking [6, 10]. However, the situation is in general more complicated [11 − 13].

[1] For early works on the possibility of TeV-scale extra dimension, see [1].

[2] For early works on this topic see [7].

[3] In the six-dimensional case the sequestering mechanism does not seem to apply [8]. One can have a nonzero gravity mediation contribution to the soft mass at the tree level, in some moduli stabilisation mechanism [8].

It has been shown recently that in S^1/Z_2 and $S^1/(Z_2 \times Z_2')$ orbifolds, higher derivative operators are dynamically generated, already at the one-loop level, as counterterms to the mass of the scalar field (usually identified with the Higgs field) [12, 13]. This happens whether the scalar field is a brane field or a zero-mode of a bulk field, and it was shown to be present regardless of the way supersymmetry was broken:[4] local (F-term) breaking, (non-local) discrete and continuous Scherk-Schwarz mechanism or additional orbifolding (Z_2'). In fact the presence of such operators has little or no dependence on the particular choice of the supersymmetry breaking mechanism, and is actually due to the number of bulk fields involved in the interaction at the loop level. Therefore, the presence of higher derivative operators is generic in theories with extra dimensions. Their implications for studies of (electroweak) symmetry breaking, for the UV behaviour of these theories, and for the hierarchy problem in particular, must then be carefully investigated. Our previous findings in [12, 13] suggested that the initial supersymmetry of the higher dimensional theory cannot prevent the emergence of such operators at the radiative level. This also raises intriguing questions on the role of initial supersymmetry in ensuring a mild UV "running" of the loop-corrected mass of the scalar (Higgs) field at scales of order $1/R^2$ where such operators become relevant.

The presence of higher derivative counterterms to the masses of scalar fields is closely related to the number of Kaluza-Klein towers to sum over in the dimensional reduction of the action at the loop level. This number is increased in the case of loop corrections from (brane-localised) interactions which do not respect momentum conservation in the extra dimensions. For a fixed order in perturbation theory this makes more likely the generation of such operators from (brane localised) Yukawa interactions than from (bulk) gauge interactions. For similar reasons higher derivative operators also emerge from compactification as counterterms to the gauge kinetic terms [14].

These findings are ultimately related to the non-renormalisable character of the higher dimensional theories which becomes manifest above the compactification scale(s). Although supersymmetric, such (effective) theories still have some of the shortcomings of the non-renormalisable theories, such as unknown UV behaviour, controlled by the coefficients of higher dimension operators. For the case of higher (dimension) derivative operators more complications arise due to the introduction of extra degrees of freedom (ghost fields), possible unitarity violation, non-locality effects, which made such theories less popular in the past [15].

This paper aims to extend the validity of the above findings about higher derivative counterterms to the scalar field mass in one-dimensional orbifolds [12, 13], to the case of two dimensional orbifolds. For a $\mathcal{T}^2/Z_2$ compactification we shall consider the effects of the localised Yukawa interaction at the orbifold fixed point and also of the (bulk) gauge interaction on the one-loop correction to the mass of a scalar field. In many applications this field plays the role of the Higgs field. The conclusion is that both Yukawa and gauge interactions generate — at the one-loop level — higher derivative counterterms to the mass of the

[4]There is an exception in the case of F-term breaking, where a footprint of this breaking mechanism is still manifest as a coefficient in front of the higher derivative operators, but these are nevertheless generated [13].

scalar field. This indicates that, although little investigated in the past, higher derivative counterterms are, rather interestingly, a generic presence in orbifold compactifications, at the quantum level, and are not protected by non-renormalisation theorems.

The paper is organised as follows. In the next section we outline the setup of the model that we are considering. Then we derive the Kaluza-Klein spectrum of the bulk fields (vector multiplets and hypermultiplets) under the boundary conditions of the orbifold and of the Scherk-Schwarz supersymmetry breaking on $\mathcal{T}^2/Z_2$. In the presence of this breaking section 4 provides the details of the calculation of the one-loop correction to the mass of the scalar field induced by the localised Yukawa and (bulk) gauge interactions. The conclusions are presented in section 5. The appendix provides the results of evaluating series of integrals generic in orbifold compactifications, which may be useful in other applications as well.

2. The model setup.

We consider a two dimensional compactification on the $\mathcal{T}^2/Z_2$ orbifold. The two-torus is parametrised by $z, \bar{z}$ with $z = (y_1 + iy_2)/2$ and $y_1 \in (-\pi R_1, \pi R_1]$, $y_2 \in (-\pi R_2, \pi R_2]$, and is invariant under

$$z \to z + (m + nU)\, \pi R_1 \tag{2.1}$$

where m, n are integers and $U = (R_2/R_1)\, e^{i\theta} \equiv U_1 + iU_2$ is the complex structure of $\mathcal{T}^2$. The geometric action of parity is $Z_2 : z \to -z$. Therefore there appear four fixed points which are: $z = 0$, $\pi R_1/2$, $\pi R_1 U/2$ and $\pi R_1(1+U)/2$.

On the orbifold $\mathcal{T}^2/Z_2$ one can consider vector multiplets and hypermultiplets. A vector multiplet is described in a 4D language as made of a vector superfield $V(\lambda^1, A_\mu, D^3 - F_{56})$ and an adjoint chiral superfield $\Sigma((A_6 + iA_5)/\sqrt{2}, \lambda^2, D^1 + iD^2)$, where $\lambda_{1,2}$ are Weyl fermions, A_μ, A_5, A_6 the bulk gauge fields and $D^i (i = 1, 2, 3)$ the auxiliary fields. The hypermultiplet contains two chiral superfields $\Phi(\phi, \psi, F_\Phi)$ and $\Phi^c(\phi^c, \psi^c, F_{\Phi^c})$ with opposite SM quantum numbers, and where ϕ, ϕ^c are complex scalars; ψ, ψ^c are Weyl fermions and F_Φ, F_{Φ^c} are the auxiliary fields. We consider now the following parity assignments

$$\begin{aligned} \Phi(x,-z) &= \Phi(x,z)\,, & V(x,-z) &= V(x,z)\,, \\ \Phi^c(x,-z) &= -\Phi^c(x,z)\,, & \Sigma(x,-z) &= -\Sigma(x,z), \end{aligned} \tag{2.2}$$

where Φ is a bulk field. Unlike the case of the gauge multiplet which is always a bulk field, in an orbifold compactification not all SM matter fields Q, U, D, L, E or Higgs fields are necessarily bulk fields. Thus Φ may stand only for a subset of these fields and this subset will be detailed shortly. As a result of eq. (2.2) the original 6D N=1 supersymmetry is broken and the fixed points of the orbifold have a remaining 4D N=1 supersymmetry.

Let us consider that the model has a gauge symmetry G. The action for a hypermultiplet Φ belonging to a representation of the gauge group G is [16]

$$\mathcal{L}_{\text{hyper}} = \int dy_1\, dy_2 \Big\{ \int d^4\theta \left[\overline{\Phi}\, e^{2V}\, \Phi + \overline{\Phi}^c\, e^{-2V}\, \Phi^c \right] + \Big[\int d^2\theta\ \Phi^c\, (-\partial + \sqrt{2}\Sigma)\, \Phi + h.c. \Big] \Big\} \tag{2.3}$$

with $V = V^a\, T_R^a$ and $\Sigma = \Sigma^a\, T_R^a$ with T_R^a the group generators. Since not all fields of the model are bulk fields, we need the action for a brane chiral multiplet Ψ charged under the group G, which is

$$\mathcal{L}_{\rm chiral} = \int dy_1\, dy_2\, \delta(y_{1,2}) \int d^4\theta\, \overline{\Psi}\, e^{2V}\, \Psi \tag{2.4}$$

with $\delta(y_{1,2}) \equiv \delta(y_1)\, \delta(y_2)$ with $\delta(y_i)$ the one-dimensional Dirac delta function.

A generic presence for realistic model building in such compactification is a superpotential interaction, which can only be localised

$$\mathcal{L}_Y = \int dy_1\, dy_2\, \delta(y_{1,2}) \left\{ \int d^2\theta \Big[\lambda_t\, Q\, U\, H_u + \lambda_b\, Q\, D\, H_d + \cdots \Big] + \text{h.c.} \right\}. \tag{2.5}$$

The 6D coupling $\lambda_t = f_{6,t}/M_*^n = \mathcal{V}^{n/2} f_{4,t}$ where $f_{6,t}$ ($f_{4,t}$) is the dimensionless 6D (4D), M_* is the UV cutoff of the theory and $\mathcal{V} = (2\pi)^2 R_1 R_2 \sin\theta$ is the area of the underlying two-torus. Dimensional analysis gives that $n = 1$ if there are two brane fields and one bulk field in (2.5). One has $n = 2$ if there is one brane field in this equation, with the other two as bulk fields.

In the following we shall consider the one-loop effects of this superpotential interaction on the mass of the scalar component $\phi_{H_{u,d}}$ of $H_{u,d}$. One-loop corrections from gauge interactions will also be computed. For simplicity, we assume $H_{u,d}$ are brane fields. (This is no special restriction: they can also be bulk fields, in which case they respect condition (2.2) for hypermultiplets. In that case the correction to the mass of the scalar field will refer to the zero-mode of $\phi_{H_{u,d}}$). Before proceeding with the calculation, one must address, for realistic model building, the breaking of the remaining 4D N=1 supersymmetry. This is considered below, with its implications on the spectrum of the bulk fields of the model.

3. Scherk-Schwarz breaking of supersymmetry on $\mathcal{T}^2/Z_2$.

We shall use the continuous Scherk-Schwarz mechanism [17] on $\mathcal{T}^2/Z_2$ orbifold with complex structure U for the underlying two-torus, in order to break the remaining supersymmetry of the fixed points. One can also consider other methods of breaking such as the discrete version of the Scherk-Schwarz mechanism,[5] but we expect to obtain similar conclusions.

On the orbifold $\mathcal{T}^2/Z_2$ the orbifold boundary conditions and the Scherk-Schwarz twists of the bulk gaugino $\lambda \equiv (\lambda^1, \lambda^2)^T$ are as follows,

$$Z_2 : \quad \lambda(x, -z) = \sigma_3\, \lambda(x, z) \equiv P\lambda(x, z), \tag{3.1}$$

$$T_1 : \quad \lambda(x, z + \pi R_1) = e^{-2\, i\pi\, \omega_1\, \sigma_2} \lambda(x, z) \equiv T_1\, \lambda(x, z), \tag{3.2}$$

$$T_2 : \quad \lambda(x, z + \pi R_1 U) = e^{-2\, i\pi\, \omega_2\, \sigma_2} \lambda(x, z) \equiv T_2\, \lambda(x, z) \tag{3.3}$$

where ω_1, ω_2 are real (arbitrary) parameters. Here we note that the consistency conditions $T_i\, P\, T_i = P$, $(i = 1, 2)$ and $T_1\, T_2 = T_2\, T_1$ are satisfied. The study of these boundary

[5] See for example details in [9].

conditions on the action for the gauginos is easier if we introduce the untwisted fields χ defined by

$$\lambda(x,z) = e^{-i(\bar{\alpha}\, z + \alpha\, \bar{z})\sigma_2}\, \chi(x,z), \qquad \text{with} \qquad \alpha = \frac{1}{iR_1U_2}(\omega_1 U - \omega_2)\,. \tag{3.4}$$

One can show that χ satisfies the same orbifold boundary condition as in eq. (3.1), but unlike λ, χ is periodic on the torus. With this re-definition of the fields, we can write the gaugino kinetic term in terms of the untwisted fields χ

$$\mathcal{L} = \sum_{j=1,2} \Big(i\, \chi^j \sigma^\mu \partial_\mu \bar{\chi}^j + i \bar{\chi}^j \bar{\sigma}^\mu \partial_\mu \chi^j \Big) + \Big[-\chi^1 \partial_z \chi^2 + \chi^2 \partial_z \chi^1 + c.c. \Big] + \mathcal{L}_m \tag{3.5}$$

where $\mathcal{L}_m$ corresponds to the bulk mass terms given by

$$\mathcal{L}_m = -\Big[\bar{\alpha}\, (\chi^1 \chi^1 + \chi^2 \chi^2) + c.c. \Big]. \tag{3.6}$$

From the action given in eq. (3.5) we derive the equations of motion for gauginos

$$\begin{aligned} i\sigma^\mu \partial_\mu \bar{\chi}^2 + \partial_z \chi^1 - \bar{\alpha}\, \chi^2 &= 0\,, \\ i\bar{\sigma}^\mu \partial_\mu \chi^1 - \bar{\partial}_z \bar{\chi}^2 - \alpha\, \bar{\chi}^1 &= 0\,. \end{aligned} \tag{3.7}$$

Solving the above equations gives the solution for the untwisted gaugino as

$$\begin{pmatrix} \chi^1 \\ \chi^2 \end{pmatrix} (x,z) = \frac{1}{\sqrt{\mathcal{V}}} \sum_{n_1,n_2 \in \mathbf{Z}} \begin{pmatrix} \cos(\bar{c}_{n_1,n_2} z + c_{n_1,n_2} \bar{z}) \\ \sin(\bar{c}_{n_1,n_2} z + c_{n_1,n_2} \bar{z}) \end{pmatrix} \eta^{(n_1,n_2)}(x) \tag{3.8}$$

where $\mathcal{V}$ is the area of underlying $\mathcal{T}^2$ and

$$c_{n_1,n_2} = \frac{1}{iR_1U_2}\,(n_1 U - n_2) \tag{3.9}$$

and $i\, \sigma^\mu\, \partial_\mu\, \bar{\eta}^{(n_1,n_2)}(x) = M_{n_1,n_2} \eta^{(n_1,n_2)}(x)$. The mass spectrum is given by

$$\overline{M}_{n_1,n_2} = \frac{1}{i\, R_1 U_2} \Big[(n_1 + \omega_1) U - (n_2 + \omega_2) \Big], \tag{3.10}$$

which, if $U = i\frac{R_2}{R_1}$, simplifies into

$$\overline{M}_{n_1,n_2} = \frac{\omega_1 + n_1}{R_1} + i \bigg(\frac{\omega_2 + n_2}{R_2} \bigg). \tag{3.11}$$

Finally, using relation (3.4), we find the solution for the twisted gaugino λ

$$\begin{pmatrix} \lambda^1 \\ \lambda^2 \end{pmatrix} (x,z) = \frac{1}{\sqrt{\mathcal{V}}} \sum_{n_1,n_2 \in \mathbf{Z}} \begin{pmatrix} \cos[(\bar{c}_{n_1,n_2} + \alpha) z + (c_{n_1,n_2} + \alpha) \bar{z}] \\ \sin[(\bar{c}_{n_1,n_2} + \alpha) z + (c_{n_1,n_2} + \alpha) \bar{z}] \end{pmatrix} \eta^{(n_1,n_2)}(x)\,. \tag{3.12}$$

Eqs. (3.10), (3.11) and (3.12) will be used shortly in section 4.2 to compute the gauge corrections to a brane scalar field.

A similar mechanism can be considered for the scalars (ϕ, ϕ^c) belonging to the hypermultiplet Φ. Unlike their fermionic partners, they can acquire (under translation along $y_{1,2}$) a continuous Scherk-Schwarz phase due to the $SU(2)_R$ symmetry. Thus (ϕ, ϕ^c) respect conditions similar to (3.2), (3.3). The Scherk-Schwarz phase "lifts" the mass of their Kaluza-Klein modes, and in particular of their zero-modes which (unlike their fermionic partners) become massive, to break the remaining 4D N=1 supersymmetry. Following closely the same steps as for the gaugino fields, the mode expansion of scalars is obtained

$$\begin{pmatrix} \phi \\ \phi^{c\dagger} \end{pmatrix} (x, z) = \frac{1}{\sqrt{\mathcal{V}}} \sum_{n_1, n_2 \in \mathbf{Z}} \begin{pmatrix} \cos[(\bar{c}_{n_1,n_2} + \alpha) z + (c_{n_1,n_2} + \alpha)\bar{z}] \\ \sin[(\bar{c}_{n_1,n_2} + \alpha) z + (c_{n_1,n_2} + \alpha)\bar{z}] \end{pmatrix} \phi_{n_1,n_2}(x) \tag{3.13}$$

where $(\Box - |m_{\phi,n_1,n_2}|^2)\phi_{n_1,n_2}(x) = 0$ and

$$|m_{\phi,n_1,n_2}|^2 = \frac{(2\pi)^2}{\mathcal{V}\, U_2} \left|(n_2 + \omega_2) - U(n_1 + \omega_1)\right|^2 \tag{3.14}$$

This equation will be used in section 4.1 when computing one loop corrections which involve the mass of the Kaluza-Klein modes of the bulk scalar fields (squarks).

4. Higher derivative counterterms on the $\mathcal{T}^2/Z_2$ orbifold.

4.1 One-loop mass correction from Yukawa interaction.

In this section we compute the one-loop correction induced by interaction (2.5) to the two point Green function of the self-energy of the scalar field ϕ_{H_u} (hereafter denoted simply ϕ_H) of H_u, which is considered a brane field. The calculation is very similar if this field is a bulk field instead, and then ϕ_H and its mass correction will refer to the zero mode component. We restrict the calculation to the first interaction in (2.5), with a similar approach for the down-type interaction. We also consider that in eq. (2.5) the quark SU(2) doublets Q are bulk fields while the quark singlets U are brane fields. This apparent restriction is made to simplify the one-loop calculation we perform, to avoid the proliferation of a large number of associated Kaluza-Klein sums. It is obvious that the effects we find and which are ultimately due to the presence of two Kaluza-Klein summations, will also apply when the U fields are also bulk fields[6] (when further Kaluza-Klein sums are present). Therefore the restriction above does not affect the generality of our results. Finally, interactions of type bulk-boundary-boundary fields that are considered here are rather standard in string theory [19, 20].

The split multiplet for the quarks is compatible with orbifold GUT scenarios. For example, in the 6D version of SU(5) case [21], only the quark doublet comes from a bulk **10** as a massless mode.[7] On the other hand, in SO(10) case [23], one has massless Q and

[6] Although it is possible to allow in the bulk only the Q (but not singlets) of one generation, both types of quark (doublet/singlet) of at least another generation must also live in the bulk for anomaly cancellation [18].

[7] This possibility is considered for the 2^{nd} generation quark doublet to explain the s-μ puzzle in 5d case [22].

L in $(4,2,1)$ under the Pati-Salam group from two bulk **16**'s. In this case, there would appear the localised anomalies with split multiplets which should be cancelled by a localised Green-Schwarz mechanism [24].

In the on-shell formulation, the interaction in eq. (2.5) for the Higgs field ϕ_H becomes

$$\begin{aligned} -\mathcal{L}_Y = & \sum_{k,l\in\mathbf{Z}} \Big[f_{4,t}\, m_{\phi^c_Q,k,l} \eta^{F_Q}_{k,l}\, \phi^{c\,\dagger}_{Q,k,l} \phi_U \phi_H + \text{h.c.} \Big] + \sum_{k,l\in\mathbf{Z}} f^2_{4,t}\, (\eta^{F_Q}_{k,l})^2 \phi^\dagger_U \phi_U \phi^\dagger_H \phi_H + \\ & + \sum_{k,l,m,n\in\mathbf{Z}} f^2_{4,t}\, \eta^{\phi_Q}_{k,l} \eta^{\phi_Q}_{m,n} \phi^\dagger_{Q,k,l} \phi_{Q,m,n} \phi^\dagger_H \phi_H + \\ & + \sum_{k,l\in\mathbf{Z}} \Big[f_{4,t}\, \eta^{\psi_Q}_{k,l}\, \psi_{Q,k,l} \psi_U \phi_H + \text{h.c.} \Big]. \end{aligned} \tag{4.1}$$

In the above equation $\eta^{F_Q}_{m,n} = \eta^{\phi_Q}_{m,n} = \eta^{\psi_Q}_{m,n} = 1$. For a Scherk-Schwarz breaking of supersymmetry, the spectrum of the squark doublet $\phi_{Q,m,n}$, $\phi^c_{Q,m,n}$, is that found in eq. (3.14)

$$|m_{\phi_Q,m,n}|^2 = |m_{\phi^c_Q,m,n}|^2 = \frac{(2\pi)^2}{\mathcal{V}\, U_2} \left| (n+\omega_2) - U(m+\omega_1) \right|^2, \qquad m,n \in \mathbf{Z}, \tag{4.2}$$

For the fermionic modes, the spectrum is

$$m^2_{\psi_Q,m,n} = \frac{(2\pi)^2}{\mathcal{V}\, U_2}\, |n - U\, m|^2, \qquad m,n \in \mathbf{Z}, \tag{4.3}$$

since they do not acquire any $\mathrm{SU}(2)_R$ Scherk-Schwarz twist.

With these considerations we can evaluate the one-loop correction to $m_{\phi_H}(q^2)$ for non-zero external momentum q^2. As mentioned, if H_u is a bulk field the correction below refers instead to the mass of the zero mode of ϕ_H. In the component formalism, there are four one-loop Feynman diagrams contributing to the Higgs mass as shown in figure 1. In the euclidean space, the bosonic and fermionic contributions are respectively

$$\begin{aligned} -\, i m^2_{\phi_H}(q^2)\Big|_{\mathcal{B}} &= -i\, f^2_{4,t} N_c\, \mu^{4-d} \sum_{m,n\in\mathbf{Z}} \int \frac{d^d p}{(2\pi)^d} \frac{(p+q)^2 + p^2}{((p+q)^2 + |m_{\phi_Q,m,n}|^2)\; p^2}, \\ -i m^2_{\phi_H}(q^2)\Big|_{\mathcal{F}} &= i\, f^2_{4,t} N_c\, \mu^{4-d} \sum_{m,n\in\mathbf{Z}} \int \frac{d^d p}{(2\pi)^d} \frac{2p.(p+q)}{((p+q)^2 + |m_{\psi_Q,m,n}|^2)\; p^2}. \end{aligned} \tag{4.4}$$

where μ is the (finite, non-zero) mass scale introduced by the DR scheme that we use ($d{=}4{-}\epsilon$, $\epsilon{\to}0$). If $q^2{=}0$, mathematical consistency of eq. (4.4) requires the introduction of an infrared (IR) mass shift m of the Kaluza-Klein masses in the fermionic denominator, to ensure that the fermionic contribution of the *massless* mode (0,0) does not vanish in the DR scheme used here.[8] Supersymmetry then requires such IR mass shift be introduced for both fermionic and bosonic contributions, $|m_{\phi_{Q,m,n}}|^2 \to |m_{\phi_{Q,m,n}}|^2 + m^2$ and $|m_{\psi_{Q,m,n}}|^2 \to |m_{\psi_{Q,m,n}}|^2 {+} m^2$. Then, a quartic divergence m^4/ϵ can be shown to be present in bosonic and fermionic contributions to $m^2_{\phi_H}(0)$, which cancels in their sum by initial supersymmetry.[9] These arguments will become clear shortly from our more general computation of $m^2_{\phi_H}(q^2)$, presented below.

[8] This vanishing would then be an artifact of our DR regularisation.

[9] We return to this point later in the text.

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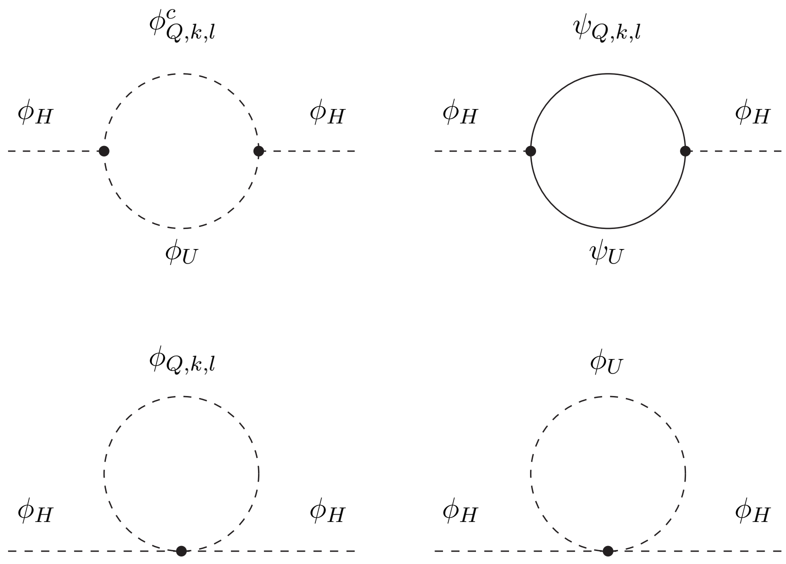

Figure 1: One-loop Feynman diagrams contributing to the Higgs mass.

After computing the momentum integrals in the DR scheme, one uses the spectrum in eqs. (4.2), (4.3) to perform the summations to find the one-loop result (with $\kappa_\epsilon \equiv (\mu\sqrt{\mathcal{V}})^\epsilon$)

$$\begin{aligned} m^2_{\phi_H}(q^2)\Big|_{\mathcal{B}} &= \frac{f^2_{4,t}\, N_c\, \kappa_\epsilon}{2\,\mathcal{V}} \int_0^1 dx \left\{ \frac{2-\epsilon/2}{\pi}\, \mathcal{J}_2[\omega_1,\omega_2,\delta] + \frac{q^2\,\mathcal{V}}{(2\pi)^2}\Big(x^2 - x + \frac{1}{2}\Big)\, \mathcal{J}_1[\omega_1,\omega_2,\delta] \right\}, \\ m^2_{\phi_H}(q^2)\Big|_{\mathcal{F}} &= -\frac{f^2_{4,t}\, N_c\, \kappa_\epsilon}{2\,\mathcal{V}} \int_0^1 dx \left\{ \frac{2-\epsilon/2}{\pi}\, \mathcal{J}_2[0,0,\delta] + \frac{q^2\,\mathcal{V}}{(2\pi)^2}\, x(x-1)\, \mathcal{J}_1[0,0,\delta] \right\} \end{aligned} \tag{4.5}$$

The functions $\mathcal{J}_p$, $(p = 1, 2)$ have the following definition and leading behaviour in ϵ

$$\begin{aligned} \mathcal{J}_p[\rho_1,\rho_2,\delta] &\equiv \sum_{n_1,n_2\in\mathbf{Z}} \int_0^\infty \frac{dt}{t^{p-\epsilon/2}}\; e^{-\pi t \left[\,\delta + \tau\,|(n_2+\rho_2) - U(n_1+\rho_1)|^2\,\right]} \\ &= \frac{(-\pi\,\delta)^p}{p\,\tau\,U_2}\left[\frac{2}{\epsilon}\right] + \mathcal{J}_p^f[\rho_1,\rho_2,\delta] + \mathcal{O}(\epsilon), \qquad \rho_{1,2}\in\mathbf{R}; \qquad p = 1, 2\,. \end{aligned} \tag{4.6}$$

where $\mathcal{J}_p^f$, $(p = 1, 2)$ are finite (no poles in ϵ) and their expression can be found in appendix A, eqs. (A.2) to (A.10). Note that the pole structure of $\mathcal{J}_{1,2}$ is *independent* of $\rho_{1,2}$, which mark the differences between bosonic and fermionic spectra eqs. (4.2), (4.3). Further, in eq. (4.5)

$$\tau \equiv \frac{x}{U_2}, \qquad \delta \equiv x\,(1-x)\frac{q^2\,\mathcal{V}}{(2\pi)^2} \tag{4.7}$$

The terms in eq. (4.5) multiplied by a q^2 coefficient are absent when considering $m^2_{\phi_H}(q^2 = 0)$, as can be seen directly from eq. (4.4) at $q^2 = 0$. In eq. (4.5) the role of the aforementioned IR mass shift (m^2) is played by the non-zero q^2. The total one-loop mass correction which is the sum of the two equations in (4.5) is then

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$$m_{\phi_H}^2(q^2) = \frac{f_{4,t}^2 N_c}{\pi\,\mathcal{V}} \int_0^1 dx \left[\mathcal{J}_2[\omega_1,\omega_2,\delta] - \mathcal{J}_2[0,0,\delta]\right] + \\ + \frac{f_{4,t}^2 N_c\,\kappa_\epsilon}{8\pi^2}\, q^2 \int_0^1 dx \left[\left(x\,(x-1)+\frac{1}{2}\right)\mathcal{J}_1[\omega_1,\omega_2,\delta] - x(x-1)\mathcal{J}_1[0,0,\delta]\right]. \quad (4.8)$$

with δ,τ as in (4.7). The second line above is not present when computing $m_{\phi_H}(q^2=0)$. Using eq. (4.6), we find that the divergence of $\mathcal{J}_2$ which is proportional to $\delta^2/\epsilon \sim q^4/\epsilon$ is cancelled[10] between the bosonic and fermionic contributions due to equal number of bosonic and fermionic degrees of freedom (ensured by the initial supersymmetry). Therefore, $m_{\phi_H}^2(0)$ given by the first line in (4.8), is finite at the one-loop level. There remains however a divergence in (4.8) from the $\mathcal{J}_1$ functions, which although have (according to (4.6)) identical pole structure (UV behaviour), they have a different coefficient (x dependence) in front. Then eq. (4.8) gives

$$m_{\phi_H}^2(q^2) = m_{\phi_H}^2(0) - \frac{f_{4,t}^2 N_c}{16\pi}\,\frac{q^4\mathcal{V}}{(2\pi)^2}\left[\frac{1}{\epsilon} + \ln\left(\mu\sqrt{\mathcal{V}}\right)\right] + \frac{1}{\mathcal{V}}\,\mathcal{O}(q^2\,\mathcal{V})\,. \quad (4.9)$$

where $\mathcal{O}(q^2\,\mathcal{V})$ are finite terms originating from the (finite part of) J_1 functions and which can be evaluated numerically using the formulae in the appendix. Also, in (4.9)

$$m_{\phi_H}^2(0) = \frac{f_{4,t}^2\,N_c}{2\pi\mathcal{V}}\Bigg\{ -\frac{2}{3}\pi^2 U_2^2 \Delta_{\omega_1}^2\,(1-\Delta_{\omega_1})^2 + \\ + \sum_{n_1\in\mathbf{Z}}\Bigg[|n_1+\omega_1|\mathrm{Li}_2(e^{-2\pi\sigma_{n_1}}) + \frac{1}{2\pi\,U_2}\mathrm{Li}_3(e^{-2\pi\sigma_{n_1}}) + \text{c.c.} - \\ - (\omega_{1,2}\to 0)\Bigg]\Bigg\} \quad (4.10)$$

where Δ_{ω_i} is the fractional part of ω_i, defined as $\Delta_{\omega_i} = \omega_i - [\omega_i]$, $0 \le \Delta_{\omega_i} < 1$ and $[\omega_i]\in\mathbf{Z}$, $i=1,2$. Finally $\sigma_{n_1} = i\,[\omega_2 - U_1(\omega_1+n_1)] + U_2\,|n_1+\omega_1|$.

To conclude, in eq. (4.9) a quartic divergence q^4/ϵ remains present at one-loop in the two point Green function of the scalar field self-energy. This is of *identical* type to that discussed above and cancelled by initial supersymmetry in $m_{\phi_H}^2(0)$ (and also referred to as m^4/ϵ). In conclusion (initial) supersymmetry does not protect against the presence of this (remaining) divergence and this finding questions the power of (initial) supersymmetry in maintaining, after compactification, a mild UV behaviour of the scalar field mass.

The presence of this divergence signals the need for a higher dimensional (derivative) counterterm. This finding is not too surprising if we recall that initial (6D) theory, although supersymmetric, is nevertheless non-renormalisable, where the presence of such operators as loop counterterms can be expected. These findings extend previous studies for the case of $S_1/(Z_2\times Z_2')$ and S_1/Z_2 orbifolds [12, 13, 25] where similar counterterms were found, regardless of the supersymmetry breaking mechanism considered there. The counterterm

[10] This divergence is nothing but the $\frac{m^4}{\epsilon}$ discussed after eq. (4.4), cancelled in $m_{\phi_H}^2(0)$ by initial supersymmetry.

which cancels the pole in (4.9) and respects the symmetries of the model is (if H_u is a brane field)

$$\int d^4x\, d^2\theta\, d^2\overline{\theta}\, \lambda_t^2\, H_u^\dagger \Box H_u \sim f_t^2 \int d^4x\ \mathcal{V}\ \phi_H^\dagger \Box^2 \phi_H + \cdots \tag{4.11}$$

If H_u is instead a bulk field, a similar result is obtained, and the counterterm is

$$\begin{aligned}\int d^4x\, dy_1\, dy_2 \int d^2\theta\, d^2\overline{\theta}\, \delta(y_1)\delta(y_2)\, \lambda_t^2\, H_u^\dagger \Box H_u &\sim f_t^2 \int d^4x\, \mathcal{V} \sum_{k,l,m,n\in\mathbf{Z}} \phi_{H,k,l}^\dagger \Box^2 \phi_{H,m,n} \\ &\sim f_t^2 \int d^4x\, \mathcal{V}\, \phi_{H,0,0}^\dagger \Box^2 \phi_{H,0,0} + \cdots \end{aligned}\tag{4.12}$$

The localisation of the counterterm is justified by a simple but powerful argument that if the counterterm were not localised, it should have N=2 supersymmetry (rather than N=1) and thus would necessarily involve the H_u^c field. However H_u^c has no Yukawa interaction, see eq. (2.5) and thus there is no bulk counterterm for (4.9).

4.2 One-loop gauge correction to a brane scalar field.

Now let us consider the gauge correction to the mass of a brane scalar ϕ_H. Using eq. (2.4) the one-loop gauge correction at zero external momentum, in the DR scheme is (see also [26])

$$m_{\phi_H}^2(0) = -4\, g_4^2\, \mu^{4-d} \bigg\{ \sum_{n_1,n_2\in\mathbf{Z}} \int \frac{d^dp}{(2\pi)^d} \frac{1}{p^2 + |M_{n_1,n_2}(\omega_1,\omega_2)|^2} - \Big(\omega_{1,2}\to 0\Big) \bigg\} \tag{4.13}$$

where $g_4 = g_6/\sqrt{\mathcal{V}},\ d{=}4{-}\epsilon$ and the Kaluza-Klein mass spectrum is

$$|M_{n_1,n_2}(\omega_1,\omega_2)|^2 = \frac{(2\pi)^2}{\mathcal{V}\, U_2}\ |(n_2+\omega_2) - U(n_1+\omega_1)|^2 \,. \tag{4.14}$$

In eq. (4.13) we must introduce a small (but otherwise arbitrary) mass shift m^2 of the denominators to avoid ambiguities specific to the DR scheme that we are using. Indeed, $M_{0,0}(0,0) = 0$ and in that case the integral in (4.13) with $\omega_i = 0$ would vanish for the $(0,0)$ Kaluza-Klein mode. A non-zero mass shift[11]

$$|M_{n_1,n_2}(\omega_1,\omega_2)|^2 \to |M_{n_1,n_2}(\omega_1,\omega_2)|^2 + m^2 \tag{4.15}$$

ensures the mathematical consistency of eq. (4.13) and the steps taken in its evaluation below. By supersymmetry, one must introduce this mass shift in both denominators in eq. (4.13). At the end of the calculation one takes $m^2\mathcal{V} \to 0$. After performing the integrals one obtains

[11] This is similar to Yukawa corrections to $m_{\phi_H}^2(0)$, see also the text before eq. (4.5), and eq. (4.10).

$$
\begin{aligned}
m^2_{\phi_H}(0) &= \frac{4\,g_4^2\,\kappa_\epsilon}{4\,\pi\,\mathcal{V}}\left\{\mathcal{J}_2\Big[0,0,\frac{m^2\mathcal{V}}{(2\pi)^2}\Big]-\mathcal{J}_2\Big[\omega_1,\omega_2,\frac{m^2\mathcal{V}}{(2\pi)^2}\Big]\right\}_{m^2\,\mathcal{V}\rightarrow 0}\\
&= \frac{4\,g_4^2}{4\,\pi\mathcal{V}}\left\{\mathcal{J}_2^f\Big[0,0,\frac{m^2\mathcal{V}}{(2\pi)^2}\Big]+\frac{m^4\mathcal{V}^2}{(4\pi)^2\,\epsilon}-\mathcal{J}_2^f\Big[\omega_1,\omega_2,\frac{m^2\mathcal{V}}{(2\pi)^2}\Big]-\frac{m^4\mathcal{V}^2}{(4\pi)^2\,\epsilon}\right\}_{m^2\,\mathcal{V}\rightarrow 0}+\mathcal{O}(\epsilon)\\
&= \frac{4\,g_4^2}{4\,\pi\,\mathcal{V}}\left\{\frac{2}{3}\,\pi^2\,U_2^2\,\Delta_{\omega_1}^2\,(1-\Delta_{\omega_1})^2-\right. \qquad (4.16)\\
&\qquad\left.-\sum_{n_1\in\mathbf{Z}}\Big[|n_1+\omega_1|\,\mathrm{Li}_2(e^{-2\pi\sigma_{n_1}})+\frac{1}{2\pi\,U_2}\mathrm{Li}_3(e^{-2\pi\sigma_{n_1}})+\text{c.c.}-(\omega_{1,2}\rightarrow 0)\Big]\right\}
\end{aligned}
$$

where in the last step a notation identical to that after eq. (4.10) was used. To obtain this result we used the definitions of $\mathcal{J}_2$ and $\mathcal{J}_2^f$ functions eq. (4.6) with $\tau = 1/U_2$ and $\delta = m^2\mathcal{V}/(2\pi)^2$. It turns out that the divergent part $m^4\,\mathcal{V}/\epsilon$ cancels out (due to initial supersymmetry) in the difference between bosonic and fermionic contributions, and one finds a finite (i.e. no poles in ϵ) one-loop correction to the mass of the brane scalar. The result in (4.16) agrees with that in ref. [9] for $U_1 = 0$.

To evaluate the momentum dependence of the gauge correction to the mass of the brane scalar field, $m^2_{\phi_H}(q^2)$, we shall use in the following the superfield formalism.[12] For this purpose we compute the gauge correction to the propagator of a (massless) brane chiral multiplet in the absence of supersymmetry breaking. To do so we need to consider only one supergraph with brane-chiral and bulk-vector multiplets "running" in the loop. We assume that, as in the 4D case, the soft (Scherk-Schwarz) breaking does not renormalize the propagator of a massless brane chiral multiplet. With an appropriate gauge fixing term, i.e. the 6D version of the super Feynman gauge, the action for the vector superfield is

$$
S_6 = \int d^4x\,dy_1\,dy_2\,d^2\theta\,d^2\bar{\theta}\,V[-\Box-\partial_5^2-\partial_6^2]V\,. \qquad (4.17)
$$

Using this action, we compute the one-loop gauge correction to the propagator of a brane chiral superfield (of 4-momentum q) and located at the origin ($y_{1,2}=0$), which equals

$$
-\frac{g_6^2}{\mathcal{V}}\int\frac{d^4q}{(2\pi)^4}\,A(q)\left[\int d^4\theta\;\overline{H}(-q,\theta)\,H(q,\theta)\right] \qquad (4.18)
$$

where

$$
\begin{aligned}
A(q) &= \mu^{4-d}\sum_{n_1,n_2\in\mathbf{Z}}\int\frac{d^dk}{(2\pi)^d}\frac{1}{(q+k)^2(k^2+M^2_{n_1,n_2}(0,0))}\\
&= \frac{\kappa_\epsilon}{(4\pi)^2}\int_0^1 dx\sum_{n_1,n_2\in\mathbf{Z}}\int_0^\infty\frac{dt}{t^{1-\epsilon/2}}\;e^{-\pi t(\delta+\tau|n_2-Un_1|^2)} \qquad (4.19)
\end{aligned}
$$

with $\epsilon = 4-d$ and

$$
\kappa_\epsilon = (\mu\sqrt{\mathcal{V}})^\epsilon,\qquad \tau=\frac{x}{U_2}\,,\qquad \delta = x(1-x)\frac{q^2\mathcal{V}}{(2\pi)^2} \qquad (4.20)
$$

[12]One could also consider the momentum dependence of gauge correction in component formalism as in [9]. However, the supersymmetric gauge fixing term would change the component field calculation. For this reason we find it easier to work in the superspace formalism for the momentum-dependent part.

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We use the expression of $\mathcal{J}_1$ of eq. (4.6) (see also appendix A), with τ, δ given in (4.20), to find

$$q^2\, A(q) = -\frac{1}{16\pi}\frac{q^4\,\mathcal{V}}{(2\pi)^2}\left[\frac{1}{\epsilon} + \ln(\mu\sqrt{\mathcal{V}})\right] + \frac{q^2}{(4\pi)^2}\int_0^1 dx\, \mathcal{J}_1^f[0,0,x(1-x)\,q^2\mathcal{V}/(2\pi)^2] + \mathcal{O}(\epsilon)\,. \tag{4.21}$$

We thus find that a divergence $q^4\,\mathcal{V}/\epsilon$ is generated in the one-loop corrected $q^2\,A(q)$ which is the coefficient (in momentum space) of the kinetic term of the brane scalar field, component of the superfield H. Therefore, a higher derivative counterterm for the brane chiral multiplet is needed in the tree-level action, to cancel the one-loop divergent term. Its structure is similar to that found in (4.11).

In conclusion, similarly to the brane-localised superpotential interactions, gauge interactions also generate, already at one-loop, higher derivative counterterms to the mass of the brane scalar field. Note that one could consider a *formal* limit $\theta \to 0$, when the two dimensions collapse onto each other. Then $\mathcal{V} = (2\pi)^2 R_1 R_2 \sin\theta$ vanishes, the quartic divergence $q^4\,\mathcal{V}/\epsilon$ present in eqs. (4.9), (4.21), is not present and no higher derivative operators are generated at one-loop for one extra dimension (although they will be generated at higher orders).[13] However, in order to keep the 6D (dimensionful) Yukawa and gauge couplings non-zero in a 5D limit, one must keep the volume non-zero (and finite), which is the limit of a "squeezed" torus. As a result the higher derivative counterterms do not decouple in the 5D limit.

5. Further remarks and conclusions.

In the context of general 6D compactifications on the orbifold $\mathcal{T}^2/Z_2$ and with Scherk-Schwarz mechanism for supersymmetry breaking we investigated the one-loop corrections to the mass of a scalar field, induced by (brane localised) superpotentials and by (bulk) gauge interactions. Our results show that both interactions usually generate, after transmission of supersymmetry breaking, higher derivative operators as one-loop counterterms to the mass of the scalar field. This is an important finding, in particular for the hierarchy problem, since in such models the scalar field can be regarded as a Higgs field candidate.

Although such operators are higher dimensional (and thus suppressed by the UV cutoff of initial theory, or in a 4D language by the volume) raising this scale suppresses them at the classical level only. Indeed, we showed that such operators are nevertheless generated dynamically, already at one-loop, while integrating out the bulk modes. As a result of this, we find that the two-point Green function for the scalar field self-energy has, above the compactification scale and under the UV scaling of the external momentum $q \to \lambda_s\, q$, a quartic dependence on the scaling parameter λ_s. This happens despite the initial supersymmetry of the higher dimensional theory and its soft (Scherk-Schwarz) breaking and raises intriguing questions one the role of initial supersymmetry in enforcing a mild UV "running" of scalar fields masses (above $1/R_{1,2}$). Our technical results can also be used

[13]This is not in contradiction with findings in 5D case [12, 13, 25] where such operators were nevertheless generated, since there Yukawa interaction involved two bulk fields (unlike here where it has one bulk field only).

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to investigate the running of the scalar field mass *across* the compactification scale, from $q^2 \ll 1/R_{1,2}^2$ to $q^2 \gg 1/R_{1,2}^2$.

The presence of such higher derivative counterterms to the scalar fields masses is in the end not too surprising if we recall that initial theory, although supersymmetric, is nevertheless non-renormalisable and in these theories such operators are expected to be generated. At the technical level, the presence of these operators is related to the number of bulk fields in the interaction. The origin of these operators is due, in our case, to a mixing effect between a *winding* zero-mode (on the lattice dual to that of Kaluza-Klein modes) wrt one dimension and the (infinite) series of Kaluza-Klein modes of the second dimension. This indicates the *non-perturbative* and non-local nature of the origin of these counterterms. In the absence of an UV completion, the unknown coefficients (in the action) of such higher derivative counterterms prevent one from making predictions about physics at (momentum) scales at/above the compactification scales, despite the supersymmetric nature of the uncompactified theory. Models with higher derivative operators may also raise potential conceptual problems, since in their presence further complications arise, such as unitarity violation, the presence of additional ghost fields or non-locality effects.

Our results extend to the case of two dimensional orbifolds, previous findings for S^1/Z_2 and $S^1/(Z_2 \times Z_2')$ compactifications where brane-localised higher derivative operators are also generated at one-loop or beyond, regardless of the details of the supersymmetry breaking mechanism considered there. We expect that our results remain valid for the case of other two-dimensional orbifolds $\mathcal{T}^2/Z_N$, $N > 2$ and other mechanisms for (the 4D) supersymmetry breaking, not considered here (like local breaking of supersymmetry). Our argument to support this uses that the origin of these operators is related to the number of bulk fields involved in the interaction and to the details of the spectrum. These can be the main differences from our $\mathcal{T}^2/Z_2$. Assuming similar interactions for $\mathcal{T}^2/Z_N$ orbifolds, the Kaluza-Klein spectrum will only have different values for the parameters $\omega_{1,2}$ in the text, but as shown, the pole structure of the functions $\mathcal{J}_{1,2}$, and thus the presence of higher derivative counterterms to the mass of the scalar field, is clearly independent of these parameters.

Higher derivative operators can have important implications for the scalar potential and the cosmological constant in theories with compact dimensions, although their exact role at the loop level was little investigated so far. Let us address some of the issues involved. It is a common approach in orbifold compactifications to investigate the one-loop corrections to the scalar field masses and their UV behaviour by computing instead the corresponding one-loop improved scalar potential $\Lambda(\phi)$, using $\Lambda(\phi) = \text{Tr} \log \det(\Box + \text{m}^2(\phi))$. Here the trace Tr is taken over all Kaluza-Klein modes associated with the compactification. A similar formula "upgraded" to respect string symmetries (such as modular invariance) and to include additional string states [27], is also the starting point for the more comprehensive string calculations. However, with this formula one can miss effects from higher derivative operators, which are *relevant* at/above the compactification scale(s). Indeed, the second derivative of such potential wrt scalar field ϕ only provides $m_\phi^2(0)$ and misses the effects of higher derivative operators which may be present in the classical action (of an otherwise non-renormalisable theory) and not accounted for in the above expression of $\Lambda(\phi)$.

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To account for this while still using the loop improved scalar potential approach, in the context of higher dimensional theories, one has to take account of higher derivative operators already in the partition function of the tree level action. In this case the above expression for one-loop $\Lambda(\phi)$ is changed and is formally given by $\Lambda(\phi) = \mathrm{Tr}\ln\det(\sigma\Box^2\mathcal{V} + \Box + \mathrm{m}^2(\phi))$ although the exact form can further depend on other details of the theory. Here σ is the (unknown) coefficient of the higher derivative operators in the tree level action. In this expression higher derivative operators provide the leading UV contribution, and thus must be included in the calculation of $\Lambda(\phi)$. The scalar field (physical) mass computed from $\Lambda(\phi)$ then includes their effects too. These observations also have implications for the one-loop cosmological constant in compactified theories.

We should mention that the higher derivative operators on the visible brane can ensure, when included in the tree level action, a better UV behaviour of the theory on the brane. This is because in their presence the propagators change and, as a result of this, loop corrections become less divergent or even finite (and this motivated in the past the use of such operators as regulators in 4D theories). However the loop corrections will then depend on the scale of such operators, which is the scale of "new" physics. The presence of higher derivative counterterms to scalar field masses after compactification of higher dimensional and *supersymmetric* theories, indicates that, in order to solve the hierarchy problem, one needs a dynamical mechanism to fix the scale of these operators.

To conclude, we argued that higher derivative operators are a common presence in compactifications of higher dimensional theories and are radiatively generated as counterterms to the scalar masses or the couplings in the theory. Their presence at the quantum level underlines the importance of their further investigation, with implications for the hierarchy and cosmological constant problems.

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A. Calculation of one-loop integrals on $\mathcal{T}_2/Z_2$.

In the text (eq. (4.6)) we used the series of regularised integrals $\mathcal{J}_p$, for $p = 1, 2$ defined as

$$\mathcal{J}_p[\rho_1, \rho_2, \delta] \equiv \sum_{n_1, n_2 \in \mathbf{Z}} \int_0^\infty \frac{dt}{t^{p+\epsilon}}\, e^{-\pi\, t\, \left[\, \delta + \tau\, |n_2 + \rho_2 - U\, (n_1 + \rho_1)|^2\, \right]}, \qquad \tau, \delta > 0;\ \rho_{1,2} \in \mathbf{R} \quad \text{(A.1)}$$

with $U \equiv U_1 + iU_2$, $U_2 > 0$, $U_1 \in \mathbf{R}$. To recover eq. (4.6) one must replace $\epsilon \to -\epsilon/2$ in (A.1).

This expression of $\mathcal{J}_p$, $p=1,2$ generalises previous results in the appendix of [13, 28]. The evaluation of $\mathcal{J}_{1,2}$ follows similar steps (see also appendix B in [29]). After a long calculation one obtains that, if $0\leq \delta/(\tau U_2^2)<1$

$$\begin{aligned}\mathcal{J}_1[\rho_1,\rho_2,\delta] &= \frac{\pi\delta}{\tau\, U_2}\bigg\{\frac{1}{\epsilon}+\ln\Big[4\pi\,(\tau U_2^2)\,e^{\gamma+\psi(\Delta_{\rho_1})+\psi(-\Delta_{\rho_1})}\Big]\bigg\}+ \qquad (A.2)\\ &\quad +2\pi U_2\Big[\frac{1}{6}+\Delta_{\rho_1}^2-\Big(\delta/(\tau U_2^2)+\Delta_{\rho_1}^2\Big)^{\frac{1}{2}}\Big]-\sum_{n_1\in\mathbf{Z}}\ln\Big|1-e^{-2\pi\,\gamma(n_1)}\Big|^2+\\ &\quad +\sqrt{\pi}\,U_2\sum_{p\geq 1}^{\infty}\frac{\Gamma[p+1/2]}{(p+1)!}\left[\frac{-\delta}{\tau U_2^2}\right]^{p+1}\Big(\zeta[2p+1,1+\Delta_{\rho_1}]+\zeta[2p+1,1-\Delta_{\rho_1}]\Big)\end{aligned}$$

while if one has $\delta/(\tau U_2^2)\geq 1$

$$\begin{aligned}\mathcal{J}_1[\rho_1,\rho_2,\delta] &= \frac{\pi\delta}{\tau\, U_2}\bigg\{\frac{1}{\epsilon}+\ln\Big[4\pi\,\delta\, e^{\gamma-1}\Big]\bigg\}-\sum_{n_1\in\mathbf{Z}}\ln\Big|1-e^{-2\pi\,\gamma(n_1)}\Big|^2+\\ &\quad +4\left[\frac{\delta}{\tau}\right]^{\frac{1}{2}}\sum_{\tilde n_1>0}\frac{1}{\tilde n_1}\cos(2\pi\tilde n_1\,\rho_1)K_1\Big(2\pi\tilde n_1(\delta/(\tau U_2^2))^{\frac{1}{2}}\Big) \qquad (A.3)\end{aligned}$$

with the notation

$$\begin{aligned}\gamma(n_1) &= \frac{1}{\sqrt{\tau}}[z(n_1)]^{\frac{1}{2}}-i(\rho_2-U_1(n_1+\rho_1))\\ z(n_1) &= \delta+\tau U_2^2(n_1+\rho_1)^2 \qquad (A.4)\end{aligned}$$

Here $\zeta[z,a]$ is the Hurwitz Zeta function, $\zeta[z,a]=\sum_{n\geq 0}(n+a)^{-z}$, Re $z>1, a\neq 0,-1,-2,\cdots$, and $\psi(x)=d/dx\,\ln\Gamma[x]$. Eqs. (A.2), (A.3) depend on fractional part of $\rho_{1,2}$ defined by $\Delta_{\rho_i}\equiv\rho_i-[\rho_i]$ with $0\leq\Delta_{\rho_i}<1$, $[\rho_i]\in\mathbf{Z}$. Finally, K_n is the modified Bessel function [30]

$$\int_0^\infty dx\, x^{\nu-1}e^{-bx^p-ax^{-p}}=\frac{2}{p}\left[\frac{a}{b}\right]^{\frac{\nu}{2p}}K_{\frac{\nu}{p}}(2\sqrt{a\,b}),\qquad Re(b),\ Re(a)>0 \qquad (A.5)$$

with

$$K_1[x]=e^{-x}\sqrt{\frac{\pi}{2x}}\left[1+\frac{3}{8x}-\frac{15}{128x^2}+\mathcal{O}(1/x^3)\right] \qquad (A.6)$$

which is strongly suppressed at large argument.

One also finds that, if $\delta\ll\tau U_2^2$ and $\delta\ll 1$

$$\begin{aligned}\mathcal{J}_1[\rho_1,\rho_2,\delta\ll 1] &= \frac{\pi\delta}{\tau\,U_2}\frac{1}{\epsilon}-\ln\left|\frac{\vartheta_1(\rho_2-U\rho_1|U)}{(\rho_2-U\rho_1)\,\eta(U)}\,e^{i\pi U\rho_1^2}\right|^2-\ln(\delta/\tau+|\rho_2-U\rho_1|^2)\\ \mathcal{J}_1[1/2,1/2,\delta\ll 1] &= \frac{\pi\delta}{\tau\,U_2}\frac{1}{\epsilon}-\ln\left|\frac{\vartheta_1(1/2-U/2|U)}{\eta(U)}\,e^{i\pi U/4}\right|^2\\ \mathcal{J}_1[0,0,\delta\ll 1] &= \frac{\pi\delta}{\tau\,U_2}\frac{1}{\epsilon}-\ln\Big[4\pi^2\,|\eta(U)|^4\,\tau^{-1}\Big]-\ln\delta \qquad (A.7)\end{aligned}$$

Above we used eq.(G-4) in [28] and the notation

$$\eta(\tau) \equiv e^{\pi i\tau/12}\prod_{n\geq 1}(1-e^{2i\pi\tau\, n}),$$
$$\vartheta_1(z|\tau) \equiv 2q^{1/8}\sin(\pi z)\prod_{n\geq 1}(1-q^n)(1-q^n e^{2i\pi z})(1-q^n e^{-2i\pi z}), \qquad q\equiv e^{2i\pi\tau} \quad \text{(A.8)}$$

In the following we also provide the result for $\mathcal{J}_2[\rho_1,\rho_2,\delta]$ whose calculation is similar. If $0\leq \delta/(\tau U_2^2)<1$

$$\begin{aligned}\mathcal{J}_2[\rho_1,\rho_2,\delta] =& -\frac{\pi^2\delta^2}{2\tau U_2}\frac{1}{\epsilon} - \frac{\pi^2\delta^2}{2\tau U_2}\ln\left[4\pi(\tau U_2^2)e^{\gamma+\psi(\Delta_{\rho_1})+\psi(-\Delta_{\rho_1})}\right] + \\ &+\pi^2\tau U_2^3\frac{1}{3}\left\{\frac{1}{15}-2\Delta_{\rho_1}^2(1+\Delta_{\rho_1}^2)-6\frac{\delta}{\tau U_2^2}\left[\frac{1}{6}+\Delta_{\rho_1}^2\right]+4\left[\frac{\delta}{(\tau U_2^2)}+\Delta_{\rho_1}^2\right]^{\frac{3}{2}}\right\}+ \\ &+\sum_{n_1\in\mathbf{Z}}\left\{(\tau\, z(n_1))^{\frac{1}{2}}\mathrm{Li}_2(e^{-2\pi\gamma(n_1)})+\frac{\tau}{2\pi}\mathrm{Li}_3(e^{-2\pi\gamma(n_1)})+\text{c.c.}\right\}+ \\ &+\pi^{3/2}\tau U_2^3\sum_{p\geq 1}^{\infty}\frac{\Gamma[p+1/2]}{(p+2)!}\left[\frac{-\delta}{\tau U_2^2}\right]^{p+2}\Big(\zeta[2p+1,1+\Delta_{\rho_1}]+\zeta[2p+1,1-\Delta_{\rho_1}]\Big) \quad \text{(A.9)}\end{aligned}$$

If instead $\delta/(\tau U_2^2)\geq 1$

$$\begin{aligned}\mathcal{J}_2[\rho_1,\rho_2,\delta] =& -\frac{\pi^2\delta^2}{2\tau U_2}\frac{1}{\epsilon} - \frac{\pi^2\delta^2}{2\tau U_2}\ln\left[\pi\,\delta e^{\gamma-3/2}\right]+ \\ &+\sum_{n_1\in\mathbf{Z}}\left\{(\tau\, z(n_1))^{\frac{1}{2}}\,\mathrm{Li}_2(e^{-2\pi\gamma(n_1)})+\frac{\tau}{2\pi}\,\mathrm{Li}_3(e^{-2\pi\gamma(n_1)})+\text{c.c.}\right\}+ \\ &+4\,\delta\, U_2\sum_{\tilde n_1>0}\frac{1}{\tilde n_1^2}\,\cos(2\pi\tilde n_1\rho_1)\,K_2\Big(2\pi\,\tilde n_1(\delta/(\tau U_2^2))^{1/2}\Big) \quad \text{(A.10)}\end{aligned}$$

where we used the notation in eq. (A.4).

In the text we used the simpler case $\delta\ll 1$, $\delta/(\tau U_2^2)\ll 1$, when

$$\begin{aligned}J_2[\rho_1,\rho_2,\delta\ll 1] =& -\frac{\pi^2\delta^2}{2\,\tau\, U_2}\frac{1}{\epsilon}+\pi^2\tau\, U_2^3\,\frac{1}{3}\left[\frac{1}{15}-2\Delta_{\rho_1}^2(1-\Delta_{\rho_1})^2\right]+ \\ &+\sum_{n_1\in\mathbf{Z}}\left\{\tau\, U_2\,|n_1+\rho_1|\,\mathrm{Li}_2(e^{-2\pi\sigma_{n_1}})+\frac{\tau}{2\pi}\,\mathrm{Li}_3(e^{-2\pi\sigma_{n_1}})+\text{c.c}\right\} \quad \text{(A.11)}\end{aligned}$$

with $\sigma_{n_1}=i\,[\rho_2-U_1(\rho_1+n_1)]+U_2\,|n_1+\rho_1|$. Also

$$\begin{aligned}J_2[0,0,\delta\ll 1] =& -\frac{\pi^2\delta^2}{2\,\tau\, U_2}\frac{1}{\epsilon}+\frac{\pi^2}{45}\,\tau\, U_2^3+ \\ &+\frac{\tau}{2\pi}\sum_{n_1\in\mathbf{Z}}\left\{2\pi\, U_2\,|n_1|\,\mathrm{Li}_2(e^{-2\pi\sigma_{n_1}})+\mathrm{Li}_3(e^{-2\pi\sigma_{n_1}})+c.c.\right\} \quad \text{(A.12)}\end{aligned}$$

with $\sigma_{n_1}=U_2|n_1|-i\,U_1\,n_1$. Finally

$$\begin{aligned}J_2[1/2,1/2,\delta\ll 1] =& -\frac{\pi^2\delta^2}{2\,\tau\, U_2}\frac{1}{\epsilon}-\frac{7\pi^2}{360}\,\tau\, U_2^3+ \quad \text{(A.13)} \\ &+\frac{\tau}{2\pi}\sum_{n_1\in\mathbf{Z}}\left\{2\pi\, U_2\,|n_1+1/2|\,\mathrm{Li}_2(-e^{-2\pi\sigma_{n_1}})+\mathrm{Li}_3(-e^{-2\pi\sigma_{n_1}})+\text{c.c.}\right\}\end{aligned}$$

with $\sigma_{n_1} = U_2|n_1 + 1/2| - iU_1(n + 1/2)$. Setting $U_1 = 0$ in this section recovers the results of the appendix in [13].

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